According to Wikipedia, if , then

Assume that:

To avoid having to retain every single and , ideally we would summarize this information for each path. This is only needed if we have many observations i within each path. Depending on the time steps of the GPS tracking device, we don’t need the approximation below.

Let’s perform a second-order Taylor series expansion around and for

If we take the expected value, we get:

Here are the derivatives:

Because of this, the final expression that we end up with is given by:

Combining everything together, we get:

The summary statistics that we need to store for each path are the first and second moments. I need to check if this equation is correct.

#----------------------------------------------------

If we assume that

Define

This implies that:

As a result, we have that:

And therefore

Notice that is log-normal. This implies that

In this expression